Modified Enlarged 18pt

OXFORD CAMBRIDGE AND RSA EXAMINATIONS

Friday 12 June 2020 – Afternoon

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension

Time allowed: 2 hours
plus your additional time allowance

YOU MUST HAVE:
the Printed Answer Booklet
the Insert
a scientific or graphical calculator

READ INSTRUCTIONS OVERLEAF

DC (CJ) 200442/2

* 9 2 0 7 6 2 4 9 3 7 *

INSTRUCTIONS

Use black ink. You can use an HB pencil, but only for graphs and diagrams.

Write your answer to each question in the space provided in the PRINTED ANSWER BOOKLET. If you need extra space use the lined pages at the end of the Printed Answer Booklet. The question numbers must be clearly shown.

Fill in the boxes on the front of the Printed Answer Booklet.

Answer ALL the questions.

Where appropriate, your answer should be supported with working. Marks might be given for using a correct method, even if your answer is wrong.

Give your final answers to a degree of accuracy that is appropriate to the context.

Do NOT send this Question Paper for marking. Keep it in the centre or recycle it.

INFORMATION

The total mark for this paper is 75.

The marks for each question are shown in brackets [ ].

ADVICE

Read each question carefully before you start your answer.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \quad \text{for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$f(x)$	$f'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v}$, $\dfrac{dy}{dx} = \dfrac{v\dfrac{du}{dx} - u\dfrac{dv}{dx}}{v^2}$

Differentiation from first principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Integration

$$\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$$

$$\int f'(x)\big(f(x)\big)^n\,dx = \frac{1}{n+1}\big(f(x)\big)^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{dv}{dx}\,dx = uv - \int v\frac{du}{dx}\,dx$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1 - \frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$$\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B} \qquad \left(A \pm B \neq (k + \tfrac{1}{2})\pi\right)$$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0 + y_n) + 2(y_1 + y_2 + \ldots + y_{n-1})\}$,
where $h = \frac{b-a}{n}$

The Newton-Raphson iteration for solving

$\mathrm{f}(x) = 0$: $x_{n+1} = x_n - \frac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$$

$$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$$

OR $\quad \mathrm{P}(A \mid B) = \frac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx}$$

where $S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r\, p^r q^{n-r}$ where $q = 1 - p$

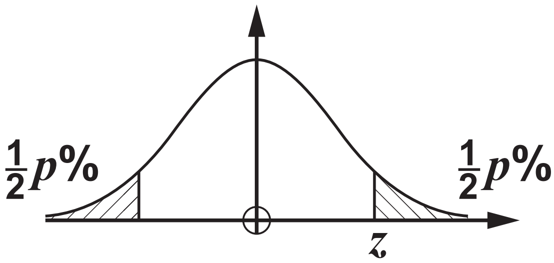
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\overline{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\overline{X} - \mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0, 1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u + v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u} + \mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

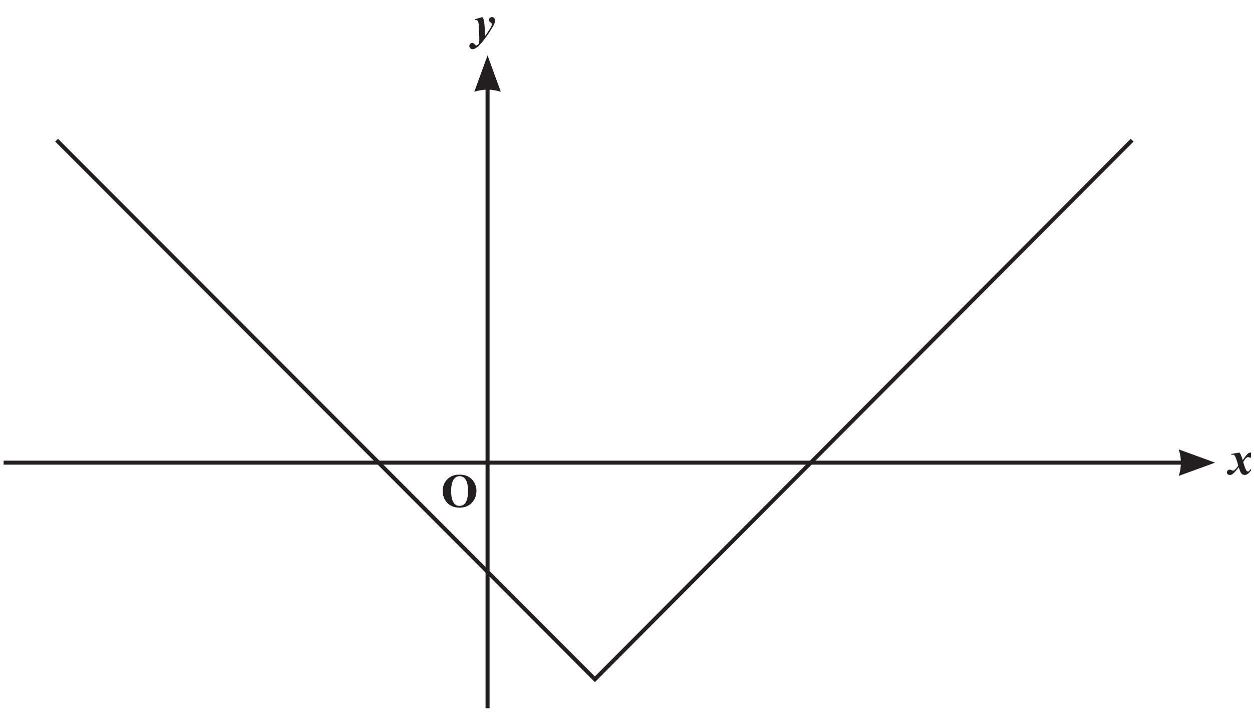
Answer ALL the questions.

SECTION A (60 marks)

1 Find the value of $\sum_{r=1}^{5} 2^r(r-1)$. **[2]**

2 The graph of $y = |1-x| - 2$ is shown in Fig. 2.

FIG. 2



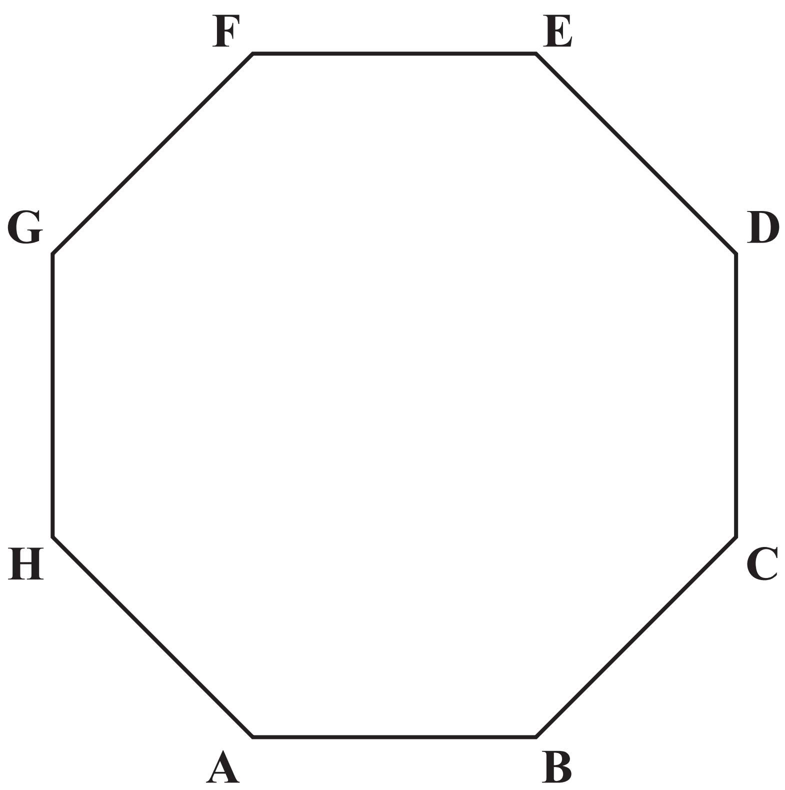
Determine the set of values of x for which $|1-x| > 2$. **[4]**

3 **A particular phone battery will last 10 hours when it is first used. Every time it is recharged, it will only last 98% of its previous time.**

Find the maximum total length of use for the battery. **[3]**

4 **Fig. 4 shows the regular octagon ABCDEFGH.**

FIG. 4

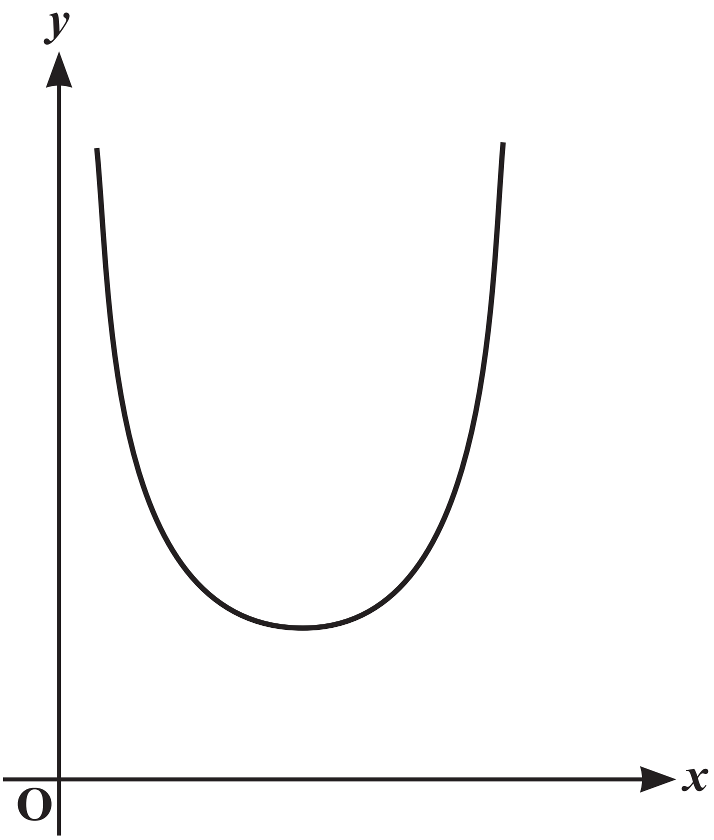


$\overrightarrow{AB} = \mathbf{i}$, $\overrightarrow{CD} = \mathbf{j}$, **where i is a unit vector parallel to the x-axis and j is a unit vector parallel to the y-axis.**

Find an exact expression for $\overrightarrow{BC}$ in terms of i and j. **[3]**

5 Fig. 5 shows part of the curve $y = \operatorname{cosec} x$ together with the x- and y-axes.

FIG. 5



(a) For the section of the curve which is shown in Fig. 5, write down

(i) the equations of the two vertical asymptotes, [2]

(ii) the coordinates of the minimum point. [1]

(b) Show that the equation $x = \operatorname{cosec} x$ has a root which lies between $x = 1$ and $x = 2$. [2]

(c) Use the iteration $x_{n+1} = \text{cosec}(x_n)$, with $x_0 = 1$, to find

(i) the values of x_1 and x_2, correct to 5 decimal places, [1]

(ii) this root of the equation, correct to 3 decimal places. [1]

(d) There is another root of $x = \text{cosec}\,x$ which lies between $x = 2$ and $x = 3$.

Determine whether the iteration $x_{n+1} = \text{cosec}(x_n)$ with $x_0 = 2.5$ converges to this root. [1]

(e) Sketch the staircase or cobweb diagram for the iteration, starting with $x_0 = 2.5$, on the diagram in the Printed Answer Booklet. [3]

6 (a) (i) Write down the derivative of e^{kx}, where k is a constant. [1]

(ii) A business has been running since 2009. They sell maths revision resources online.

Give a reason why an exponential growth model might be suitable for the annual profits for the business. [1]

Fig. 6 opposite shows the relationship between the annual profits of the business in thousands of pounds (y) and the time in years after 2009 (x). The graph of $\ln y$ plotted against x is approximately a straight line.

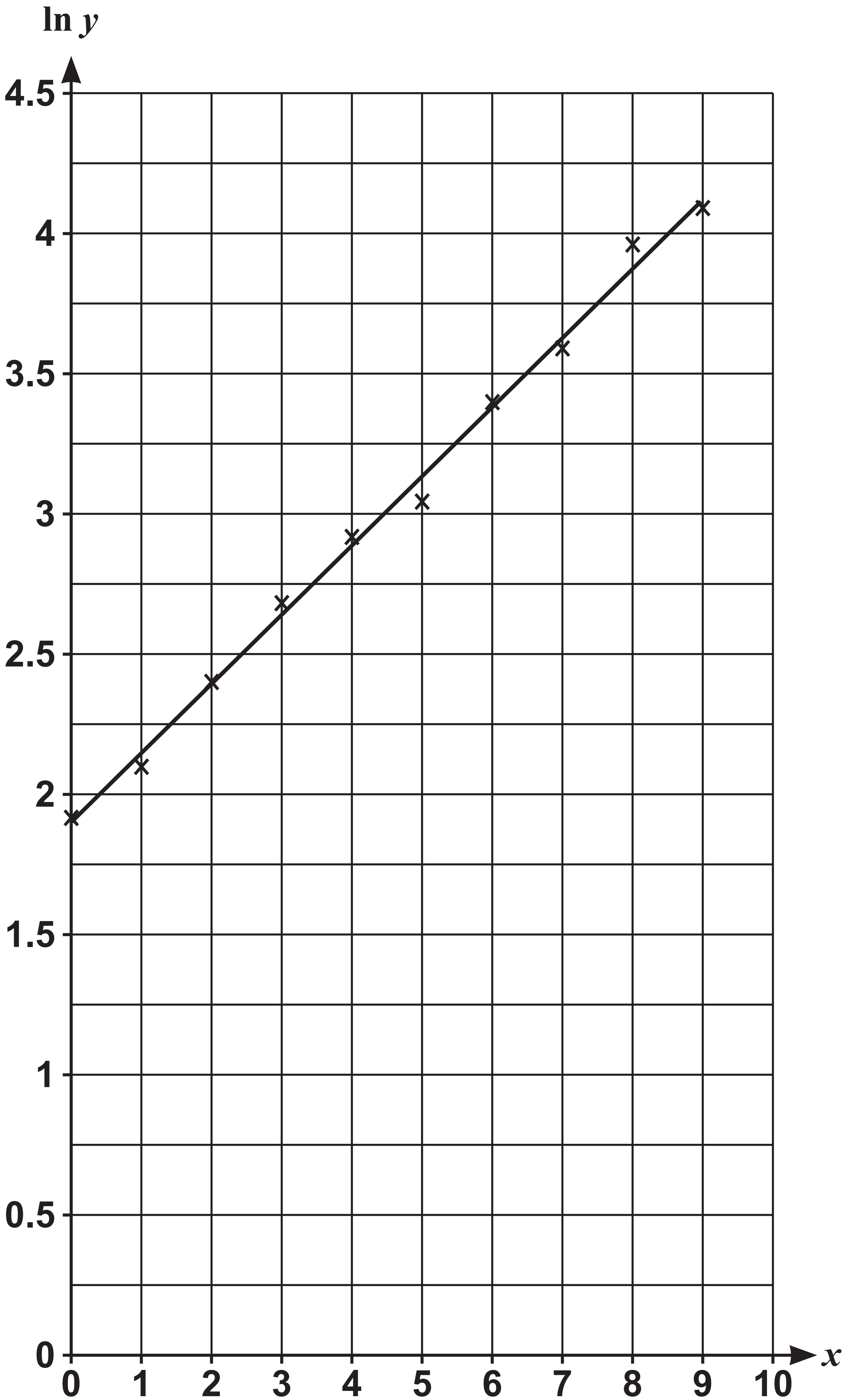
(b) Show that the straight line is consistent with a model of the form $y = Ae^{kx}$, where A and k are constants. [2]

(c) Estimate the values of A and k. [4]

(d) Use the model to predict the profit in the year 2020. [3]

(e) How reliable do you expect the prediction in part (d) to be? Justify your answer. [1]

FIG. 6



7 (a) Express $\frac{1}{x} + \frac{1}{A-x}$ as a single fraction. [1]

The population of fish in a lake is modelled by the differential equation

$$\frac{dx}{dt} = \frac{x(400-x)}{400}$$

where x is the number of fish and t is the time in years.

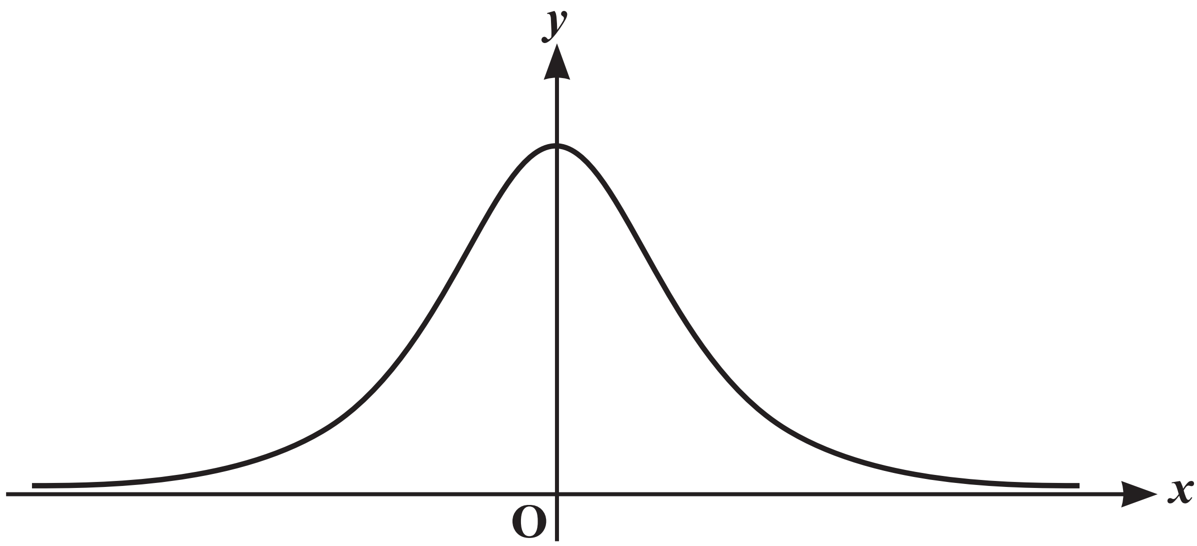
When $t = 0$, $x = 100$.

(b) In this question you must show detailed reasoning.

Find the number of fish in the lake when $t = 10$, as predicted by the model. [8]

8 (a) The curve $y = \dfrac{1}{(1+x^2)^2}$ is shown in Fig. 8.

FIG. 8



(i) Show that $\dfrac{d^2y}{dx^2} = \dfrac{20x^2 - 4}{(1+x^2)^4}$. [5]

(ii) In this question you must show detailed reasoning.

Find the set of values of x for which the curve is concave downwards. [3]

(b) Use the substitution $x = \tan\theta$ to find the exact value of $\displaystyle\int_{-1}^{1} \frac{1}{(1+x^2)^2}\,dx$. [8]

Answer ALL the questions.

SECTION B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

9 (a) Show that if $a = 1$ and $b > 1$ then $a^b < b^a$. [2]

(b) Find integer values of a and b with $b > a > 1$ and a^b not greater than b^a (a counter example to the conjecture given in lines 11–13). [1]

10 In this question you must show detailed reasoning.

Show that $\int_{e}^{\pi} \frac{1}{x}\,dx = \ln\pi - 1$ as given in line 54. [2]

11 Show that e^x is an increasing function for all values of x, as stated in line 56. [2]

12 **(a)** **Show that the only stationary point on the curve $y = \dfrac{\ln x}{x}$ occurs where $x = \mathrm{e}$, as given in lines 66–67. [3]**

(b) **Show that the stationary point is a maximum. [3]**

(c) **It follows from part (b) that, for any positive number a with $a \neq \mathrm{e}$,**

$$\frac{\ln \mathrm{e}}{\mathrm{e}} > \frac{\ln a}{a}.$$

Use this fact to show that $\mathrm{e}^a > a^{\mathrm{e}}$. [2]

END OF QUESTION PAPER

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